Advanced Algebra

Comprehensive z-score worksheet.

Name:_____

Date: _____

Solve:

1) The _____ a Normal curve is always equal to 1.

2) The mean of the **Standard Normal** curve is _____.

3) If the z-score is positive, that means _____

4) Scores of the history test were normally distributed with a mean of 83 and standard deviation of 1.6. Zachary got an 86 on the test. This corresponds to a z-score of -1.875. Explain what it means.

5) Tony is looking at a distribution with a mean of 83 and standard deviation of 26. Find the z-score of 71.65.

6) What does a z-score of 3.7 mean?

7) Butch is looking at a distribution with a mean of 155 and standard deviation of 47. Find a value with a z-score of 0.04.

8) April is looking at a distribution with a mean of 94 and standard deviation of 73. Find the z-score of 28.82.

Solve:

9) Scores of each of the previous physics tests were normally distributed with a mean of 85 and standard deviation of 4.5. Heather will be taking the test tomorrow. What is the $P(z \leq 1.78)$?

10) Scores of each of the previous history tests were normally distributed with a mean of 82 and standard deviation of 5. Floyd will be taking the test tomorrow. What is the probability that Floyd's z-score will be at least 1.4?

11) Scores of each of the previous history tests were normally distributed with a mean of 84 and standard deviation of 2.6. Adam will be taking the test tomorrow. What is the $P(0 \leq z \leq 0.38)$?

12) Scores of each of the previous algebra tests were normally distributed with a mean of 78 and standard deviation of 4.2. Ginger will be taking the test tomorrow. What is the probability that Ginger's z-score will be at least 0.71?

13) Scores of each of the previous Spanish tests were normally distributed with a mean of 76 and standard deviation of 1.8. John will be taking the test tomorrow. What is the probability of John getting between 76 and 78 on the test?

14) Scores of each of the previous algebra tests were normally distributed with a mean of 81 and standard deviation of 1.2. Ursula will be taking the test tomorrow. What is the probability of Ursula getting at least 83 on the test?

15) Scores of each of the previous biology tests were normally distributed with a mean of 78 and standard deviation of 2.5. Tyson will be taking the test tomorrow. What is the probability of Tyson getting at most 82 on the test?

16) Scores of each of the previous geometry tests were normally distributed with a mean of 83 and standard deviation of 4. Ginger will be taking the test tomorrow. What is the probability of Ginger getting between 80 and 84 on the test?

17) One notation to show mean and standard deviation is $N(\mu, \sigma)$. Scores of each of the previous algebra tests have approximated N(70, 4.9). How high must April score in order to be in the top 28%?

18) Scores of each of the previous geometry tests were normally distributed with a mean of 74 and standard deviation of 2.6. Jacob will be taking the test tomorrow. There is a 6.18% probability of Jacob getting at least what score on the test?

19) Scores of each of the previous history tests were normally distributed with a mean of 83 and standard deviation of 4.6. Ralph will be taking the test tomorrow. There is a 0.45% chance of Ralph getting at most what score on the test?

20) Scores of each of the previous English tests were normally distributed with a mean of 82 and standard deviation of 1.2. Ralph will be taking the test tomorrow. There is a 79.67% probability of Ralph getting at least what score on the test?

21) Peter is in Mrs. Kowalski's geometry class while Jacob is in Dr. Griffin's geometry class. Both teachers gave out the same test. In Mrs. Kowalski's class, the scores were normally distributed with a mean of 82 and a standard deviation of 4.8. While in Dr. Griffin's class, the scores were normally distributed with a mean of 76 and a standard deviation of 5. Peter had a z-score of 1.46 and Jacob had a z-score of 0.8. Who did better on the test?

22) Kenny is in Miss Kowalski's physics class while Ryan is in Mrs. Cox's physics class. Both teachers gave out the same test. In Miss Kowalski's class, the scores were normally distributed with a mean of 79 and a standard deviation of 3.3. While in Mrs. Cox's class, the scores were normally distributed with a mean of 83 and a standard deviation of 3.7. Kenny had a z-score of 1.52 and Ryan had a z-score of 3.24. Who did better on the test?

23) Walter is in Mrs. Brooks' calculus class while Nathan is in Mr. Bailey's calculus class. Both teachers gave out the same test. In Mrs. Brooks' class, the scores were normally distributed with a mean of 74 and a standard deviation of 1.9. While in Mr. Bailey's class, the scores were normally distributed with a mean of 76 and a standard deviation of 1.4. Walter had a z-score of -1.58 and Nathan had a z-score of -0.71. Who did better on the test?

24) Quincy is in Miss Edwards' biology class while Susan is in Dr. Bell's biology class. Both teachers gave out the same test. In Miss Edwards' class, the scores were normally distributed with a mean of 76 and a standard deviation of 3.6. While in Dr. Bell's class, the scores were normally distributed with a mean of 73 and a standard deviation of 4.1. Quincy had a z-score of 1.39 and Susan had a z-score of 1.95. Who did better on the test?

1) area under
3) $x > \mu$
5) -0.44
7) 156.88
9) 96.25%
11) 14.8%
13) 36.65%
15) 94.52%
17) 72.84
19) 70.99
21) Peter did better with a score of 89
23) Nathan did better with a score of 75

1) The _____ a Normal curve is always equal to 1 area under

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2) The mean of the Standard Normal curve is _____
For the Standard Normal curve (z curve), the mean is 0. So we get 0

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3) If the z-score is positive, that means _____ $x > \mu$

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4) Scores of the history test were normally distributed with a mean of 83 and standard deviation of -1.6. Zachary got an 86 on the test. This corresponds to a z-score of -1.875. Explain what it means.
This means that Zachary earned a score that is 1.875 standard deviations below the mean.
Furthermore, using the empirical rule, we know that over (0.5 + 0.34) = 0.84 of the students did better than Zachary.

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5) Tony is looking at a distribution with a mean of 83 and standard deviation of 26. Find the z-score of 71.65.
Z-score is the number of standard deviations away from the mean and can be calculated with the following formula:

$$z = \frac{x - \mu}{\sigma}$$

So we get z = (71.65 – 83)/(26) = -0.436538461538461
So we get: -0.44

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6) What does a z-score of 3.7 mean?
The z-score tells us the number of standard deviations we are away from the mean.
A positive z-score means that we are to the right of the mean.
So we get: the value is 3.7 standard deviations to the right of the mean.

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7) Butch is looking at a distribution with a mean of 155 and standard deviation of 47. Find a value with a z-score of 0.04.
z-score is the number of standard deviations away from the mean and can be calculated with the following formula:

$$z = \frac{x - \mu}{\sigma}$$

So we get 0.04 = (x – 155)/(47)
1.88 = x – 155
So x = 156.88

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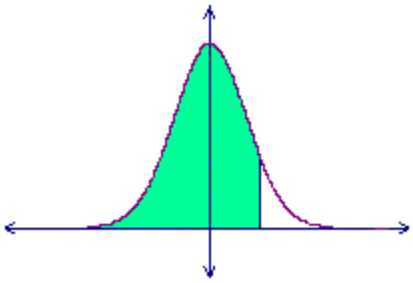
8) April is looking at a distribution with a mean of 94 and standard deviation of 73. Find the z-score of 28.82.
Z-score is the number of standard deviations away from the mean and can be calculated with the following formula:

$$z = \frac{x - \mu}{\sigma}$$

So we get z = (28.82 – 94)/(73) = -0.892876712328767, So we get: -0.89

9) Scores of each of the previous physics tests were normally distributed with a mean of 85 and standard deviation of 4.5. Heather will be taking the test tomorrow. What is the P(z ≤ 1.78)?

We are looking at a situation like:



So, we just look up z = 1.78 in the table to get 0.9625.

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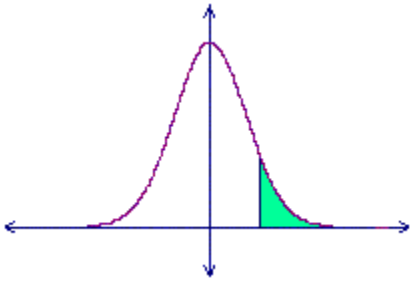
10) Scores of each of the previous history tests were normally distributed with a mean of 82 and standard deviation of 5. Floyd will be taking the test tomorrow. What is the probability that Floyd's z-score will be at least 1.4?

We are looking at a situation like:



So, we look up z = 1.4 in the table to get 0.9192.
And then we have to subtract this from 100% (a.k.a. 1). (remember, the table just gives values from -∞ to z, it doesn't give values from z to ∞).
So we end up with 0.0808.

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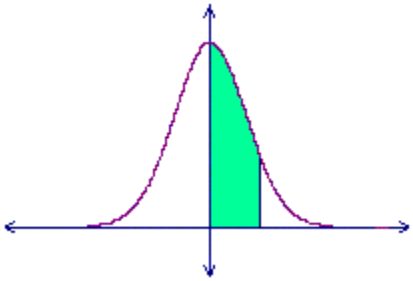
11) Scores of each of the previous history tests were normally distributed with a mean of 84 and standard deviation of 2.6. Adam will be taking the test tomorrow. What is the P(0 ≤ z ≤ 0.38)?

We are looking at a situation like:



So, we look up z = 0 in the table to get 0.5.
And we look up z = 0.38 in the table to get 0.6480.
And then we have to subtract them from each other: 0.6480 – 0.5 = 0.1480.

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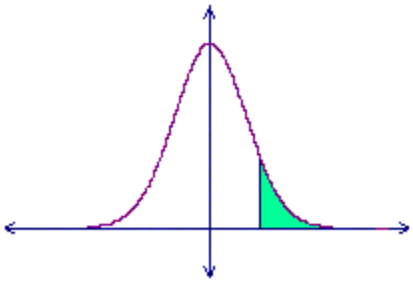
12) Scores of each of the previous algebra tests were normally distributed with a mean of 78 and standard deviation of 4.2. Ginger will be taking the test tomorrow. What is the probability that Ginger's z-score will be at least 0.71?

We are looking at a situation like:



So, we look up z = 0.71 in the table to get 0.7611.
And then we have to subtract this from 1. (remember, the table just gives values from $-\infty$ to z, it doesn't give values from z to ∞).
So we end up with 0.2389.

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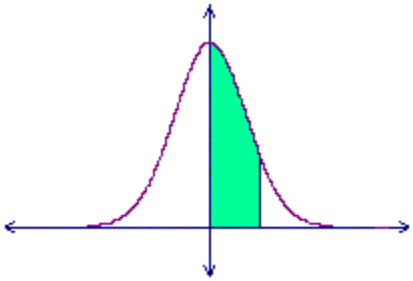
13) Scores of each of the previous Spanish tests were normally distributed with a mean of 76 and standard deviation of 1.8. John will be taking the test tomorrow. What is the probability of John getting between 76 and 78 on the test?

To calculate the z-score for 76, we'll use the formula:

$$z = \frac{x - \mu}{\sigma} = \frac{76 - 76}{1.8} = 0$$

$$z = \frac{x - \mu}{\sigma} = \frac{78 - 76}{1.8} = 1.11$$

We are looking at a situation like:



So, we look up z = 0 in the table to get 0.5.
And we look up z = 1.11 in the table to get 0.8665.
And then we have to subtract them from each other: 0.8665 – 0.5 = 0.3665.

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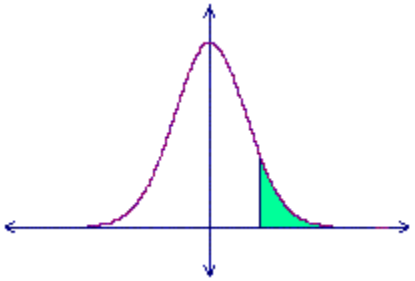
14) Scores of each of the previous algebra tests were normally distributed with a mean of 81 and standard deviation of 1.2. Ursula will be taking the test tomorrow. What is the probability of Ursula getting at least 83 on the test?

To calculate the z-score for 83, we'll use the formula:

$$z = \frac{x - \mu}{\sigma} = \frac{83 - 81}{1.2} = 1.67$$

We are looking at a situation like:



So, we look up z = 1.67 in the table to get 0.9525.
And then we have to subtract this from 1. (remember, the table just gives values from $-\infty$ to z, it doesn't give values from z to ∞).
So we end up with 0.0475.

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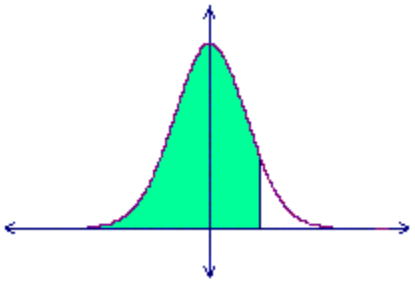
15) Scores of each of the previous biology tests were normally distributed with a mean of 78 and standard deviation of 2.5. Tyson will be taking the test tomorrow. What is the probability of Tyson getting at most 82 on the test?

To calculate the z-score for 82, we'll use the formula:

$$z = \frac{x - \mu}{\sigma} = \frac{82 - 78}{2.5} = 1.6$$

We are looking at a situation like:



So, we just look up z = 1.6 in the table to get 0.9452.

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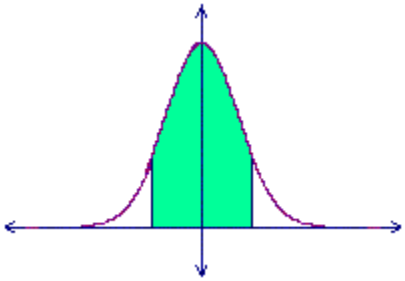
16) Scores of each of the previous geometry tests were normally distributed with a mean of 83 and standard deviation of 4. Ginger will be taking the test tomorrow. What is the probability of Ginger getting between 80 and 84 on the test?

To calculate the z-score for 80, we'll use the formula:

$$z = \frac{x - \mu}{\sigma} = \frac{80 - 83}{4} = -0.75$$

$$z = \frac{x - \mu}{\sigma} = \frac{84 - 83}{4} = 0.25$$

We are looking at a situation like:



So, we look up z = -0.75 in the table to get 0.2266.
And we look up z = 0.25 in the table to get 0.5987.
And then we have to subtract them from each other: 0.5987 – 0.2266 = 0.3721.

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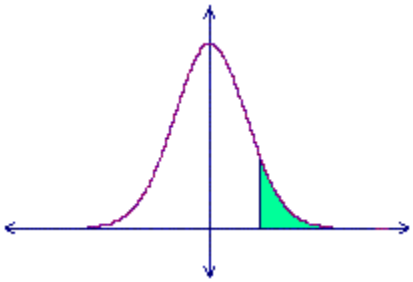
17) Scores of each of the previous algebra tests have approximated N(70, 4.9). How high must April score in order to be in the top 28%?

First, recall that N(70, 4.9) means that $\mu = 70$ and $\sigma = 4.9$.
Make sure you draw a picture. (Label everything you can). Your rough sketch should look something like this:



Next, remember that the table calculates area to the LEFT, so we need to subtract this z-score from 1 to get 0.72.
Use the table to find the closest entry to 0.72 (z = 0.58).
Next, use the definition of z to solve for x:

$$z = \frac{x - \mu}{\sigma}$$

$$0.58 = \frac{x - 70}{4.9}$$

So x = 72.84

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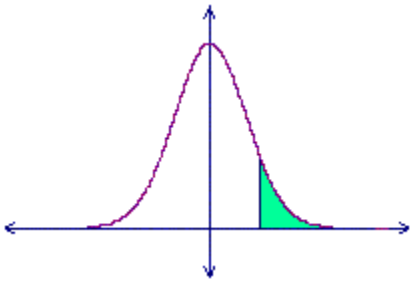
18) Scores of each of the previous geometry tests were normally distributed with a mean of 74 and standard deviation of 2.6. Jacob will be taking the test tomorrow. There is a 6.18% probability of Jacob getting at least what score on the test?

We are looking at a situation like:



So, first we have to subtract 0.0618 from 1. (remember, the table just gives values from $-\infty$ to z, it doesn't give values from z to ∞).

So, we look up p = 0.9382 in the middle of the table to find z = 1.54.

To calculate the score, we'll use the formula:

$$z = \frac{x - \mu}{\sigma}$$

$$1.54 = \frac{x - 74}{2.6}$$

$$4.004 = x - 74$$

$$78 = x$$

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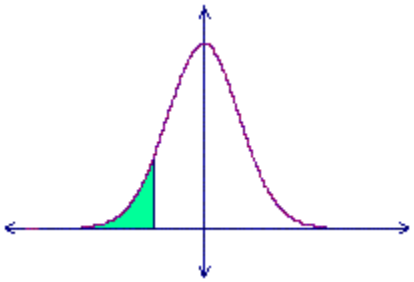
19) Scores of each of the previous history tests were normally distributed with a mean of 83 and standard deviation of 4.6. Ralph will be taking the test tomorrow. There is a 0.45% chance of Ralph getting at most what score on the test?

We are looking at a situation like:



So, we just look up 0.45% in the middle of the table to get a z-value of z = -2.61.

To calculate the score, we'll use the formula:

$$z = \frac{x - \mu}{\sigma}$$

$$-2.61 = \frac{x - 83}{4.6}$$

$$-12.006 = x - 83$$

$$70.99 = x$$

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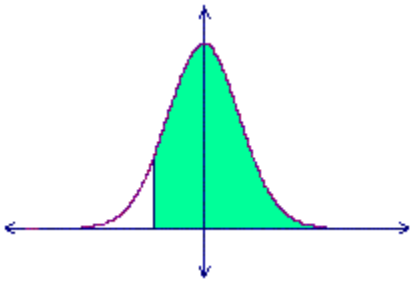
20) Scores of each of the previous English tests were normally distributed with a mean of 82 and standard deviation of 1.2. Ralph will be taking the test tomorrow. There is a 79.67% probability of Ralph getting at least what score on the test?

We are looking at a situation like:



So, first we have to subtract 0.7967 from 1. (remember, the table just gives values from $-\infty$ to z, it doesn't give values from z to ∞).

So, we look up p = 0.2033 in the middle of the table to find z = -0.83.

To calculate the score, we'll use the formula:

$$z = \frac{x - \mu}{\sigma}$$

$$-0.83 = \frac{x - 82}{1.2}$$

$-0.996 = x - 82$

$81 = x$

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21) Peter is in Mrs. Kowalski's geometry class while Jacob is in Dr. Griffin's geometry class. Both teachers gave out the same test. In Mrs. Kowalski's class, the scores were normally distributed with a mean of 82 and a standard deviation of 4.8. While in Dr. Griffin's class, the scores were normally distributed with a mean of 76 and a standard deviation of 5. Peter had a z-score of 1.46 and Jacob had a z-score of 0.8. Who did better on the test?

Let's figure out everyone's score:

Peter had a z-score of 1.46, the mean on Mrs. Kowalski's class was $\mu = 82$, and the standard deviation was $\sigma = 4.8$.

$$z = \frac{x - \mu}{\sigma}$$

$$1.46 = \frac{x - 82}{4.8}$$

$7.008 = x - 82$

$89 = x$

Jacob had a z-score of 0.8, the mean on Dr. Griffin's class was $\mu = 76$, and the standard deviation was $\sigma = 5$.

$$z = \frac{x - \mu}{\sigma}$$

$$0.8 = \frac{x - 76}{5}$$

$4 = x - 76$

$80 = x$

So Peter got a 89 and Jacob got a 80 on the test.

So we get: Peter did better with a score of 89

22) Kenny is in Miss Kowalski's physics class while Ryan is in Mrs. Cox's physics class. Both teachers gave out the same test. In Miss Kowalski's class, the scores were normally distributed with a mean of 79 and a standard deviation of 3.3. While in Mrs. Cox's class, the scores were normally distributed with a mean of 83 and a standard deviation of 3.7. Kenny had a z-score of 1.52 and Ryan had a z-score of 3.24. Who did better on the test?

Let's figure out everyone's score:

Kenny had a z-score of 1.52, the mean on Miss Kowalski's class was $\mu = 79$, and the standard deviation was $\sigma = 3.3$.

$$z = \frac{x - \mu}{\sigma}$$

$$1.52 = \frac{x - 79}{3.3}$$

$5.016 = x - 79$

$84 = x$

Ryan had a z-score of 3.24, the mean on Mrs. Cox's class was $\mu = 83$, and the standard deviation was $\sigma = 3.7$.

$$z = \frac{x - \mu}{\sigma}$$

$$3.24 = \frac{x - 83}{3.7}$$

$11.988 = x - 83$

$95 = x$

So Kenny got a 84 and Ryan got a 95 on the test.

So we get: Ryan did better with a score of 95

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23) Walter is in Mrs. Brooks' calculus class while Nathan is in Mr. Bailey's calculus class. Both teachers gave out the same test. In Mrs. Brooks' class, the scores were normally distributed with a mean of 74 and a standard deviation of 1.9. While in Mr. Bailey's class, the scores were normally distributed with a mean of 76 and a standard deviation of 1.4. Walter had a z-score of -1.58 and Nathan had a z-score of -0.71. Who did better on the test?

Let's figure out everyone's score:

Walter had a z-score of -1.58, the mean on Mrs. Brooks' class was $\mu = 74$, and the standard deviation was $\sigma = 1.9$.

$$z = \frac{x - \mu}{\sigma}$$

$$-1.58 = \frac{x - 74}{1.9}$$

$-3.002 = x - 74$

$71 = x$

Nathan had a z-score of -0.71, the mean on Mr. Bailey's class was $\mu = 76$, and the standard deviation was $\sigma = 1.4$.

$$z = \frac{x - \mu}{\sigma}$$

$$-0.71 = \frac{x - 76}{1.4}$$

$-0.994 = x - 76$

$75 = x$ So Walter got a 71 and Nathan got a 75 on the test. So we get: Nathan did better with a score of 75

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24) Quincy is in Miss Edwards' biology class while Susan is in Dr. Bell's biology class. Both teachers gave out the same test. In Miss Edwards' class, the scores were normally distributed with a mean of 76 and a standard deviation of 3.6. While in Dr. Bell's class, the scores were normally distributed with a mean of 73 and a standard deviation of 4.1. Quincy had a z-score of 1.39 and Susan had a z-score of 1.95. Who did better on the test?

Let's figure out everyone's score:

Quincy had a z-score of 1.39, the mean on Miss Edwards' class was $\mu = 76$, and the standard deviation was $\sigma = 3.6$.

$$z = \frac{x - \mu}{\sigma}$$

$$1.39 = \frac{x - 76}{3.6}$$

$$5.004 = x - 76$$

$$81 = x$$

Susan had a z-score of 1.95, the mean on Dr. Bell's class was $\mu = 73$, and the standard deviation was $\sigma = 4.1$.

$$z = \frac{x - \mu}{\sigma}$$

$$1.95 = \frac{x - 73}{4.1}$$

$$7.995 = x - 73$$

$$81 = x$$

So Quincy got a 81 and Susan got a 81 on the test.

So we get: Both people got 81 on the test.